STEPS
CAMT 2006
Let's Not Crash and Burn

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Rationale: *Students will construct various payload items, calculate volume and center of mass, and use coordinate geometry to balance the Orbiter's payload bay for a successful launch.*

TEKS Addressed:
Geometry: (b)(2)(A), (b)(7)(A), (b)(8)(D)
Additional TEKS
Algebra I: (b)(1)(B), (b)(3)(A), (b)(4)(A), (b)(5)(A), (b)(5)(B), (b)(7)(A)
Integrated Physics and Chemistry: (c)(4)(C)

Description of Activity:
- *Topic:* Center of Mass
- *Time*: three 55-minute class periods, five 55-minute class periods if extensions are included
- *Method:* student pairs (3 max)
- *Materials:*
  - Ruler, scissors, graph paper and markers
  - Triple-beam balance, manila folders and rice (for extension)
  - Graphing calculator
  - Computation Table
  - Payload Bay Grid
  - Balance Payload Bay Grid, several pennies, glue and 35mm film canister (for extension)
  - Packing the Payload: Algebra Extension worksheet (for extension)
  - Payload Bay Vocabulary Word Search (for extension)
- *Content*
  The mass of the payload affects the location of the center of mass (c.m.) of the space shuttle. The orbiter payload bay is 60-ft long and 15-ft in diameter. Equipment must be located in the payload bay in such a way that the c.m. of the entire shuttle fits within certain limits, a matter of inches. If the c.m. is off, the shuttle will crash.
- *Procedure:*
  - Have students create the base of each payload item using graph paper and cut them out (I grid-length = 1 ft on graph paper). The coordinates of the center of the object should be marked *(find the c.m. of the base of the prisms using the midpoint formula; for each cylinder, find the midpoint of the diameter of the base).* The dimensions of the figure, and the mass should be labeled on each object. (Extension: have students calculate the mass of each object instead of just giving them the mass).
  - ❖ Hand out the *Payload Bay Grid**. Label the lower left corner of the rectangle "(0, 0)". This will be the origin of the coordinate axis for the following steps (only this location will result in all measurements being positive). Label the x-axis to 38 and the y-axis to 11.
  - ❖ Lay two dimensional payload shapes inside the rectangle. No shape should be outside the 11 X 38 rectangle, nor should any shape overlap another. The objective is to create an arrangement of pieces that will have a combined c.m. near the center of the 11 X 38 rectangle at the coordinate (19, 5.5). Assign each group 2 or 3 payload items to work with or let them choose.
  - ❖ Hand out *Computation Table** and explain to students that the center of mass for the entire payload bay must be centered at (19, 5.5).
    - The center of mass (c.m.) of an individual object is located at the **center of the object**. That is the point that students will use for the object's location on the computation table
    - Show students how to use the table.
      - Enter the coordinates for the center of each base into the x and y columns of the computation table. Also enter the mass of each piece in the m column.
      - Multiply the mass by the x-coordinate and enter this information into the mx column. Multiply the mass by the y-coordinate and enter this information into the my column.
      - Find the sum of the m, mx and my columns. These can be labeled $\sum m$, $\sum mx$, and $\sum mx$.
      - Divide $\sum mx$ by $\sum m$. This will give the x-coordinate of the combined c.m.
      - Divide $\sum my$ by $\sum m$. This will give the y-coordinate of the combined c.m.
      - For a successful packing job, $cm_x = 19 \pm 1$ unit and $cm_y = 5.5 \pm 1$. If the package met these limits, the task is finished. If not, the cargo shapes should be rearranged and the computations repeated.

* Visit http://www.pasadenaisd.org/sohohs/math.htm (*Math in Action* section) for online copies of all materials (.pdf format)

Important Questions/Assessment Ideas:

- How would you construct a scale model for a geometric shape?
- How would you find the midpoint of a line segment?
- How do you find the center of a two-dimensional object? Does it matter which line segment is used (diagonal or side)?
- How do you find the coordinates for the vertices and the center of the bases of each payload item on the grid?
- For a successful packing job, $cm_x = 19 \pm 1$ unit; what does ± 1 unit mean? What if it is changed to ± .5 unit?
- If the astronauts "harvested" a piece of space debris (various dimensions and mass), could the cargo bay be reconfigured to bring it home? How should it be reconfigured?
- How would the placement of cargo be affected by the type of cargo being loaded and the need for quick accessibility? For example: On day one, the astronauts are doing an experiment with grubs. Do you really want to pack the grubs in the back of the payload bay? If an item must be readily accessible, but its placement near the front of the bay destabilizes the c.m., how could you compensate for the mass, and move the c.m. back to its proper point? You can use the movie, *Apollo 13*, as an example (the moon rocks they did not have an opportunity to pick-up - they never go a chance to land on the moon - affected their flight-line on re-entry, how did they compensate for this problem?)

Extensions:

- Add some extra shapes or duplicates to make the task more challenging.
- Shift the origin to the c.m. of the shape (19, 5.5). The inclusion of negative numbers will increase the challenge.
- Have students construct 3-dimensional cargo items, based on the given dimensions, using manila folders.
    - The third dimension of the payload bay has a depth of 8 units.
    - Extend the computation table to include the z-coordinate (z and mz columns).
    - Cm_z is computed by dividing $\sum mz$ by $\sum m$. A good solution will have a $cm_z = 4 \pm 1$ unit.
- The teacher can construct a larger model of the Payload Bay Grid that is balanced on a 35MM film canister (located under(19, 5.5)).
    - Larger model should be laminated and glued onto foam-board for extra support. (NOTE: Pennies will need to be glued under the nose of the model to balance the model on the film canister.)
    - Proportional mass will need to be calculated for the model cargo items. Using a triple-beam balance, fill each 3-D representation with rice to the appropriate weight. (use a scale of 1 cm = 1 ft for constructions; use a scale of 1g = 1kg, using rice as filler, for weight).
    - Students can bring 3-dimensional cargo items to the teacher's larger, balanced grid and test their data. (NOTE: Larger Payload Bay should be constructed using 1 cm^2 grid paper.)
- Work through the *Packing the Payload: Algebra Extension** worksheet
- Work the *Payload Bay Vocabulary Word Search**

Payload Cargo Items: (Mass given for construction of 3-D models as part of an extension exercise)

- Rectangular Prism: Base: 2 ft X 15 ft, Height: 2 ft, Mass: 35 kg
- Rectangular Prism: Base: 3 ft X 6 ft, Height: 3 ft, Mass: 45 kg
- Rectangular Prism: Base: 4 ft X 7 ft, Height: 4 ft, Mass: 98 kg
- Rectangular Prism: Base: 5 ft X 8 ft, Height: 5 ft, Mass: 128 kg
- Rectangular Prism: Base: 2 ft X 15 ft, Height: 2 ft, Mass: 189 kg
- Cylinder: Radius: 2 ft, Height: 6 ft, Mass: 62 kg
- Cylinder: Radius: 3.5 ft, Height: 4 ft, Mass: 134 kg
- Hexagonal Prism: Regular Hexagon with side of 3 ft, Height: 6 ft, Mass: 126 kg

Computation Table

Object	x	y	m	mx	my
		Sums (Σ)			
			Center of Mass		

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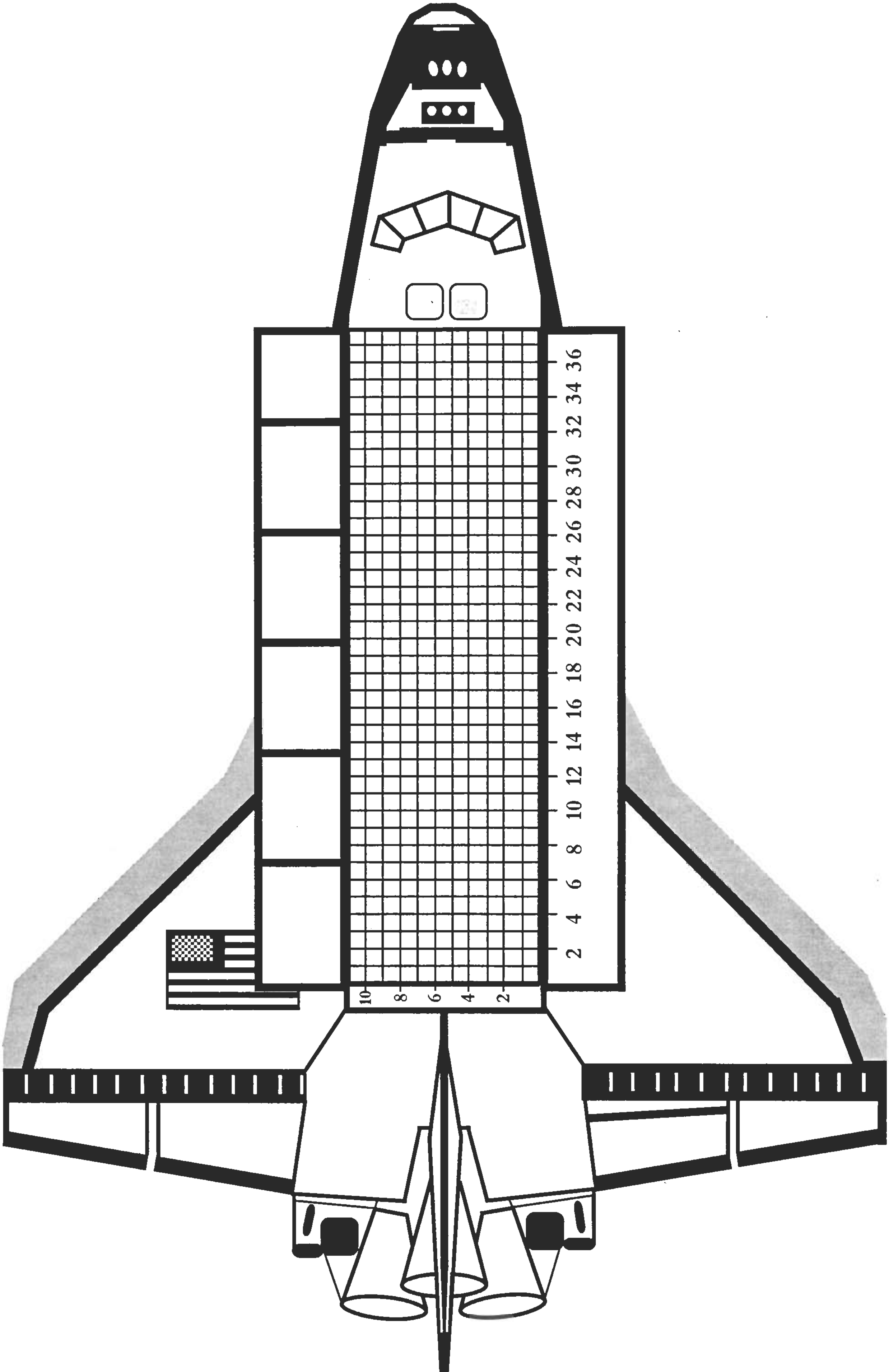

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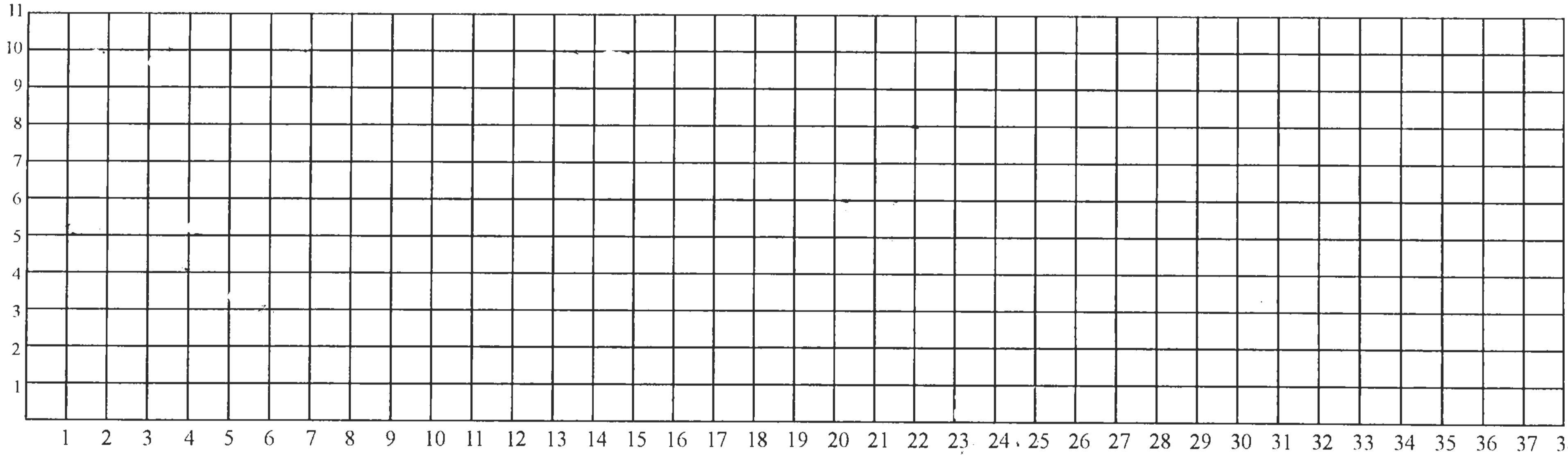
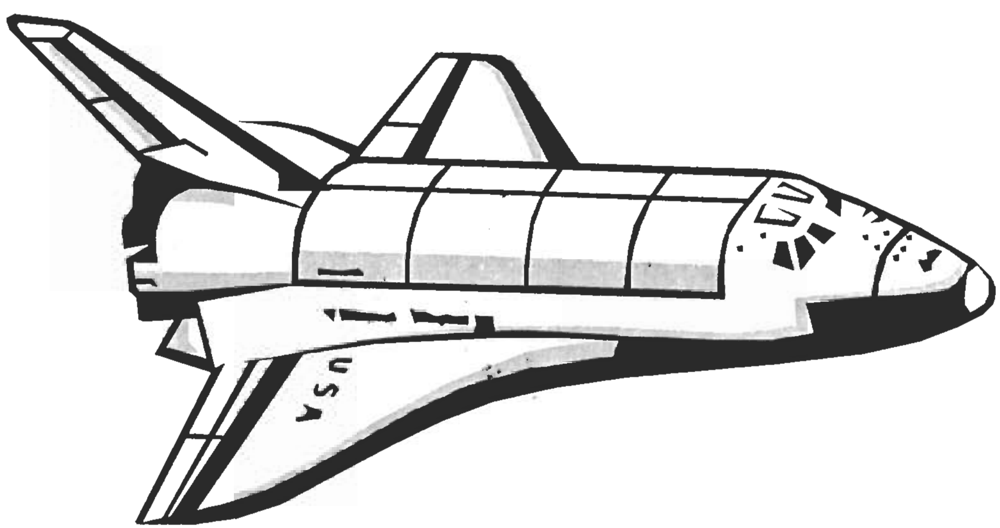
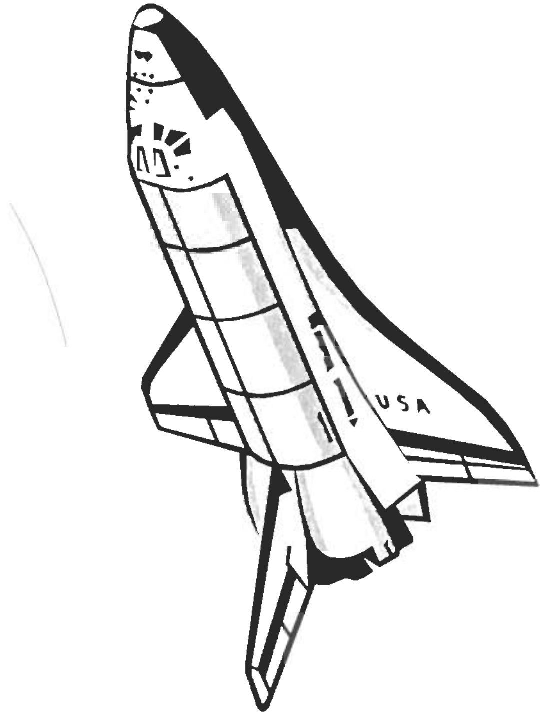
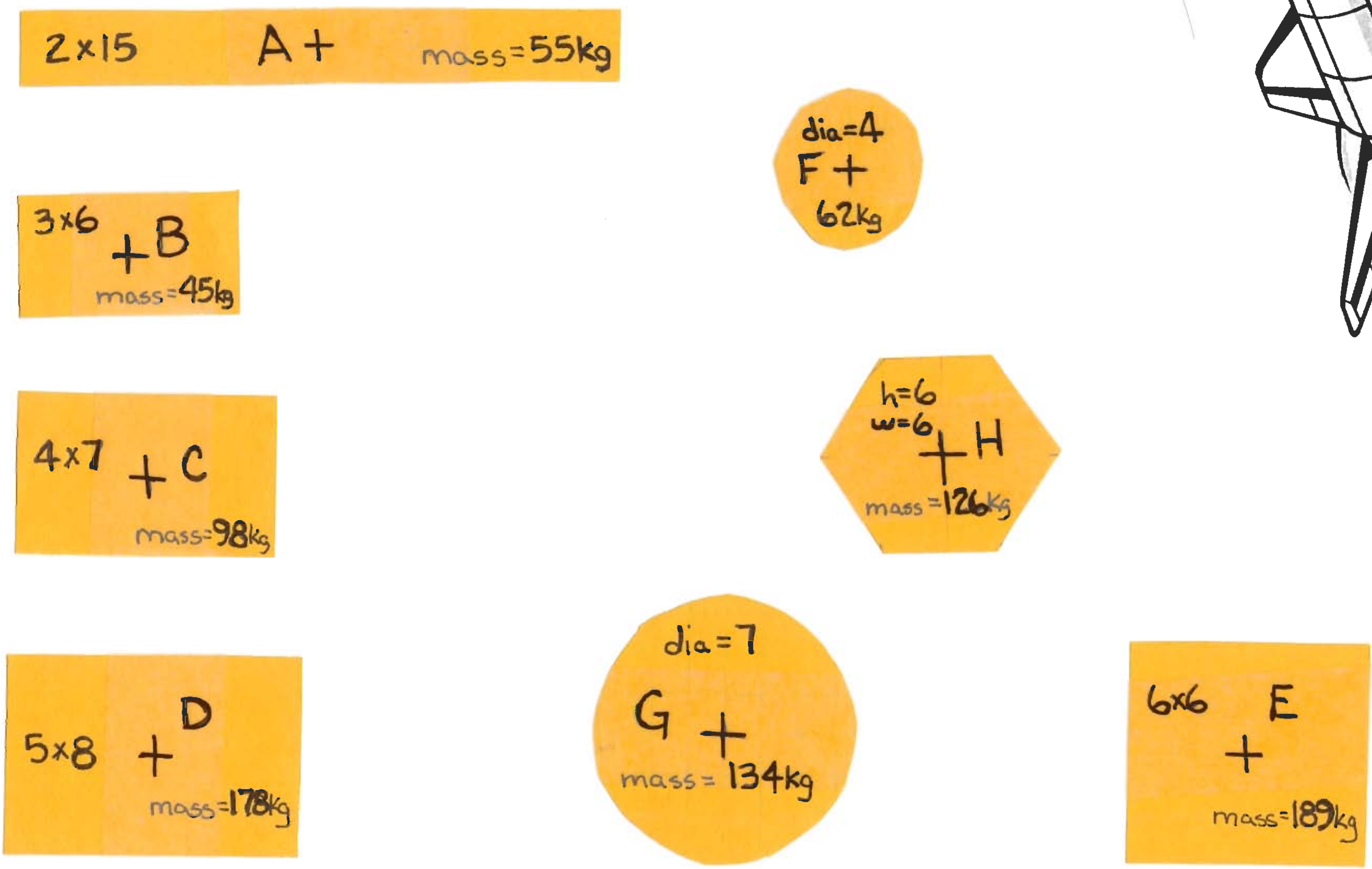
Payload Bay Grid





Cargo Shapes





Computation Tables

Object	x	y	m	mx	my
		Sums			
			Center of Mass		

Object	x	y	m	mx	my
		Sums			
			Center of Mass		

Object	x	y	m	mx	my
		Sums			
			Center of Mass		

Object	x	y	m	mx	my
		Sums			
			Center of Mass		

Computation Table

Object	x	y	m	mx	my
		Sums Σ			
			Center of Mass		

Packing the Payload: Algebra Extension

In the next part of this activity, we will use an algebraic equation to solve the same problem with only two payloads.

Object	x	y	m	mx	my
		Sums			
			Center of Mass		

1. Pick two of your original payloads and record their masses in the table above.
2. Since both of your payloads are symmetrical, what ***same*** y-coordinate could be chosen for both of masses that would allow the shuttle to balance in the y-direction? ____
3. Record your answer from number 3 in the table above and compute the center of mass for the y-coordinate.
4. Now you only need to determine the x-coordinates that would balance the shuttle orbiter. Since those two x-coordinates are unknown, use a and b as variables. Record those in the table above.
5. Using a and b for our x-coordinates, continue to fill in the table above for mx just like you did for my. Write the expression that would represent the center of mass for mx below.

6. To have a perfect center of mass, your x-coordinate would have to be at exactly 19 (the x-coordinate for the center of our payload bay). Write an equation to represent this.
7. Solve your equation in number 6 for a. Show your work below.

8. Use a graphing utility to graph your equation and sketch the graph below. Choose a friendly window appropriate for the problem situation and record your window settings. Label the x- and y-intercepts.

Xmin:

Xmax:

Xscl:

Ymin:

Ymax:

Yscl:

9. Would all of the points on the line work as x-values for locations of payloads in the payload bay? Why or why not? ______________________________

____

____

____

____

10. In the given table, record 5 different solutions for locations your payloads could be placed. Keep in mind the dimensions of the payload bay.

Object 1	Object 2
(, 5.5)	(, 5.5)
(, 5.5)	(, 5.5)
(, 5.5)	(, 5.5)
(, 5.5)	(, 5.5)

NASA Unit

Vocabulary Words

P	A	Y	L	O	A	D	H	E	U	E	N	G	K	P
D	N	K	H	V	K	F	Q	R	Q	P	N	R	K	Q
W	E	T	W	S	P	B	S	H	T	O	W	A	P	Y
L	F	N	Q	A	W	U	S	G	I	E	S	V	Z	M
L	J	L	S	E	E	C	K	T	A	L	O	I	M	N
O	W	R	I	I	E	M	U	L	O	V	D	T	Z	R
R	Z	G	K	R	T	B	K	S	O	K	V	Y	T	S
K	H	R	A	X	I	Y	E	T	B	H	Q	K	W	U
T	V	F	H	R	X	S	F	R	N	W	B	Q	M	T
I	K	U	T	C	D	G	O	V	V	B	U	S	O	Q
V	A	S	G	R	T	W	W	E	V	Y	S	F	T	P
E	I	B	X	C	V	I	U	Q	G	A	R	N	T	J
D	Y	A	W	K	Q	A	P	W	M	D	V	N	K	R
D	F	Z	M	J	Q	N	I	S	H	S	S	T	O	X
E	L	R	M	E	O	F	H	S	K	L	K	A	Q	N

DENSITY
DISTRIBUTION
GRAVITY
MASS
PAYLOAD
PITCH
ROLL
VOLUME
WEIGHT
YAW

NASA Unit
Vocabulary Words

Yaw	A position of the shuttle when the nose goes left and the tail goes right (or vice-versa).
Pitch	A position of the shuttle when the nose comes up and the tail goes down (or vice-versa).
Roll	A position of the shuttle when one wing moves up and the other down.
Payload	The load carried by an aircraft or spacecraft consisting of things (as passengers or instruments) necessary to the purpose of the flight
Distribution	The act of placing or positioning so as to be properly apportioned over or throughout an area
Weight	The force with which a body is attracted toward the earth by gravitation and which is equal to the product of the mass and the local gravitational pull
Mass	The property of a body that is a measure of its inertia and that is commonly taken as a measure of the amount of material it contains and causes it to have weight in a gravitational field
Volume	The amount of space occupied by a three-dimensional object as measured in cubic units (as quarts or liters) : cubic capacity
Density	The distribution of a quantity (as mass) per unit usually of space (as length, area, or volume)
Gravity	The attraction of the mass of the earth, the moon, or a planet for bodies at or near its surface